Report on Internship

Gravitational Waves in Effective Fields of Gravity

Intern

Revaz Beradze

Supervisor

Adam Falkowski

University Paris-Saclay / IJCLab

07.10.2021 - 29.12.2021

Introduction: With this report I intend to explain the purpose of my internship. The paper describes the work that has been carried out during my visit, the results that have been obtained so far and the future prospects of the research in this field. In the conclusion, I summarize how useful the visit was for my professional development, the experience and skills I have acquired and how it contributes to my PhD thesis.

Location and duration: My internship named ”Gravitational waves in effective fields of gravity“ was conducted at University Paris-Saclay/IJCLab under supervision of Adam Falkowski. Internship started on 7th of October and ended on 29th of December.

PhD thesis theme and topic of the internship: In order to discuss the work carried out during my internship, that will be included in my PhD thesis, I briefly introduce the topic of my PhD study.

My PhD thesis named ”Some Cosmological and Astrophysical Implications of Mirror World Model“ mainly concerns the recent detection of gravitational waves and its analysis in the context of particular dark matter model - the mirror matter model. All normal matter constitutes only 5% in the whole energy of the universe, while 68% is thought to be dark energy and 27% is elusive dark matter. The existence of dark matter is sensed only by its gravitational effects and no 'dark matter particle' has been detected so far. Nevertheless, there are lot of theories that can explain dark matter and one of them is mirror matter model.

The standard model of particle physics unifies three fundamental forces of nature, but overlooks forth fundamental force - gravity. Therefore, it fails to explain dark matter. Mirror world model is an effective field theory suggesting that all particles of standard model have their twin mirror partners, which are invisible for ordinary particles, but can interact via gravitational force. If mirror particles are real, they can form stars, galaxies and even black holes and, thus, they can make up the dark matter. To prove this theory, one should search for the effects of mirror sector in cosmological and astrophysical experiments.

A recent detection of gravitational waves was a huge discovery that served as the beginning of a new era in the field of physics. Very compact objects such as neutron stars and black holes, existing in binary systems, merge and produce gravitational waves that were captured by detectors (LIGO and Virgo). But such cataclysmic events must be accompanied by the emission of electromagnetic radiation as well. However, only one out of 90 gravitational-wave events detected so far, had accompanying electromagnetic signal. Additionally, binary system's merger rates calculated from these discoveries, were relatively high than that, expected in various models.

The objective of my thesis is to explore whether these gravitational waves can be emerged from the mirror world. A good starting point is that, as already mentioned, mirror particles are invisible and interact with our world only through gravity, correspondingly, if compact objects that emitted these gravitational waves were made of mirror particles, all kind of radiation, except gravitational, will be unnoticed for us. Besides, mirror matter is dark matter candidate and it can be about 5 times more abundant than ordinary matter. What's more, mirror world is thought to be colder, that means more stars were formed at earlier stage of the evolution of the universe. Considering all this, unexpectedly high merger rates of binary systems can be naturally explained within mirror world scenario.

My PhD thesis will be based on the papers [1–3] written together with my supervisor Assoc. Prof. Merab Gogberashvili, as part of my PhD studies at Ivane Javakhishvili Tbilisi State University. In our works, we build up the model explaining experimental data of gravitational waves within the framework of the mirror world. In our papers, we focus on the formation mechanisms of binary systems that emit gravitational waves captured by detectors. We argue, that absence of electromagnetic radiation and estimated values of binary merger rates are better explained if one considers the formation of binary systems in the mirror world model.

In the work carried out under supervision of Adam Falkowski during my internship at IJCLab, we focused on different aspect of this probable scenario. Our goal is to study the waveforms of gravitational waves and search for the possible sign of mirror world impact in the data recorded by the detectors. Our interest is aimed towards studying the systems consisting of one normal star and one mirror star. Taking into account the probability of oscillations between neutrons and mirror neutrons, existence of such systems is plausible.

In binary neutron star systems, two stars can rotate around each other for billions of years until they crash producing gravitational waves and forming a heavier neutron star or a black hole. While being far enough from each other, their motion can be described as motion of point-mass objects, i.e. their size can be neglected; this is a so-called inspiral phase. As they rotate, separation distance between the stars decreases continuously and at some point their size and internal structure starts to matter and one needs to consider tidal effects when describing the motion of the system. The effects of stars internal structure is imprinted in the gravitational waveform and exploring waves can give access to interior of the star. If one assumes that one of the stars is mirror neutron star, then situation changes: normal matter and mirror matter does not 'see' each other (except their gravitational attraction), that means, stars actually 'touch' each other at later moment than expected and it should have an impact on the gravitational waveform. Our core aim is to figure out how normal binary star inspiral waveform gets modified if one of the stars is mirror star and we want to check if this effect can be spotted in the experimental data of merging events.

Work conducted during the internship: In order to determine what type of correction emerges in a binary merger waveform if one object is a mirror star, one should know how the waveform comes out in normal case. So, the first part of my internship was devoted to studying the theory of emission of gravitational waves. Besides studying corresponding chapters in some well-known textbooks about gravitation, I thoroughly analysed the paper "The basic physics of the binary black hole merger GW150914" by LIGO/Virgo collaboration [4], which presents a simplified approach to gravitational wave data analyses, that is adequate for our purposes. To be able to reproduce the known results for binary mergers, I also employed the lecture notes by Utah State University [5].

With the purpose to get a more global view about gravitational wave physics I looked through the review articles *"Gravitational waves from merging compact binaries: How accurately can one extract the binary's parameters from the inspiral waveform?* " by Curt Cutler and Eanna E. Flanagan [6] and *"Probing the equation of state of neutron star matter with gravitational waves from binary inspirals in light of GW170817: a brief review"* by Andreas Guerra Chaves and Tanja Hinderer [7].

To get deeper insight into latest tools and modern approaches of gravitational waves physics, I researched many articles, among which the most useful for me were the following papers [8, 9]. Besides, I was lucky enough to attend **"The Paris Saclay AstroParticle Symposium 2021"** [10], that was held in the Paris-Saclay University during my visit. The symposium lasted for six weeks, where the last two weeks were completely dedicated to gravitational wave physics. This served as a great opportunity to listen to talks by highly-respected scientists of the field from all around the world and get introduced with them in person.

Equipped with all the useful knowledge, I started setting up a numerical simulation in the first

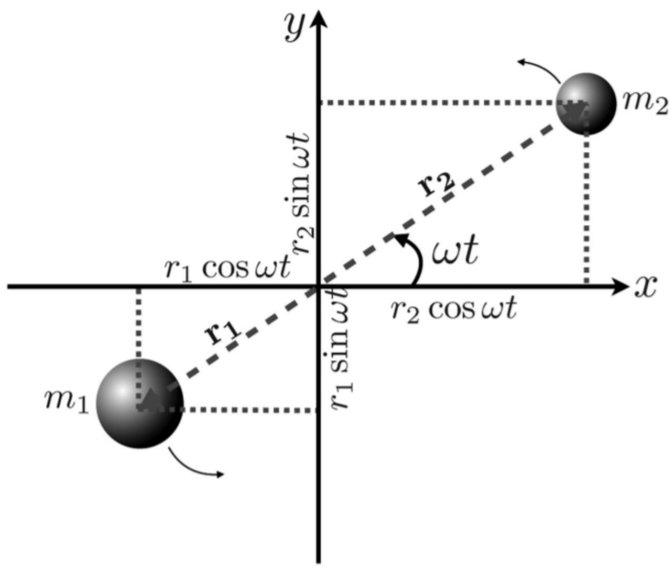

Figure A1 A two-body system, m_1 and m_2 orbiting in the xy-plane around their C.O.M.

Figure 1: The figure is adopted from [4]

place to repeat the well-known results for normal binary star mergers, before moving to a case of normal star - mirror star coalescence. In the first approximation gravitational wave emission is calculated for the inspiral of point-like objects. To get a clear vision, we are dealing with two point-mass objects that are orbiting around their center-of-mass in $X-Y$ plane, as shown in the figure (1).

Gravitational radiation is a so-called quadrupole radiation and in Einstein's theory it is given by the formula

$$Q_{ij} = \int dxdydz \; f(x,y,z) \; \rho(x,y,z) \; , \tag{1}$$

where the function f is

$$f(x,y,z) = x_i x_j - \frac{1}{3} r^2 \delta_{ij} \quad (x_i = (x,y,z), \quad r = r_1 + r_2, \quad \delta_{ij} \text{ is Kronecker's delta}) \tag{2}$$

and Dirac's Delta functions δ are used to describe a mass density distribution function for point masses

$$\rho(x,y,z) = m_1 \delta(x-x_1)\delta(y-y_1)\delta(z) + m_2\delta(x-x_2)\delta(y-y_2)\delta(z) \; . \tag{3}$$

Usage of Dirac's Delta functions guarantees that the mass distribution function is zero everywhere except one point, where the whole mass of the object is concentrated. One needs to integrate a formula for the quadrupole moment (1), which is an easy task for point-like case. The result of integration is:

$$Q_{ij} = \frac{1}{2}\mu r^2 \begin{pmatrix} \frac{1}{3} + \cos 2\omega t & \sin 2\omega t & 0 \\ \sin 2\omega t & \frac{1}{3} - \cos 2\omega t & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix} , \tag{4}$$

where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is reduced mass, ω is a frequency of rotation and t is time. The so-called gravitational wave strain tensor or a waveform for the binary system at a distance d_L is given by

$$h_{ij} = \frac{2G}{c^4 d_L}\frac{d^2 Q_{ij}}{dt^2} = \frac{4G\mu}{c^4 d_L} r^2\omega^2 \begin{pmatrix} -\cos 2\omega t & -\sin 2\omega t & 0 \\ -\sin 2\omega t & \cos 2\omega t & 0 \\ 0 & 0 & 0 \end{pmatrix} , \tag{5}$$

where G is the Newton's constant of gravitation and c is the speed of light. The rate at which energy is carried away by the gravitational waves equals to

$$\frac{dE_{GW}}{dt} = \frac{c^3}{16\pi G}\int\int \sum_{i,j=1}^{3} \frac{dh_{ij}}{dt}\frac{dh_{ij}}{dt} dS = \frac{1}{5}\frac{G}{c^5}\sum_{i,j=1}^{3}\frac{d^3 Q_{ij}}{dt^3}\frac{d^3 Q_{ij}}{dt^3} = \frac{32}{5}\frac{G}{c^5}\mu^2 r^4 \omega^6 . \tag{6}$$

Orbital energy of the system is given as

$$E_{\text{orb}} = -\frac{GM\mu}{2r} \quad (M = m_1 + m_2 \text{ is a total mass}) \tag{7}$$

and the loss of energy through gravitational radiation drains it, giving an equation

$$\frac{dE_{\text{orb}}}{dt} = \frac{GM\mu}{2r^2}\frac{dr}{dt} = -\frac{dE_{GW}}{dt} . \tag{8}$$

Assuming that the energy radiated away over each orbit is small compared to E_{orb}, one can describe each orbit as approximately Keplerian. Using Keplers third law and its time derivative

$$r^3 = \frac{GM}{\omega^2} , \qquad \dot{r} = -\frac{2}{3}\frac{\dot{\omega}}{\omega} r , \tag{9}$$

and inserting (6) in (8), one gets the main formula for a frequency change

$$\dot{\omega} = \frac{96}{5}\frac{G^{5/3}}{c^5}\mu M^{2/3}\omega^{11/3} = \frac{96}{5}(G\mathcal{M}_c)^{5/3}\omega^{11/3} , \tag{10}$$

where $\mathcal{M}_c = (\mu^3 M^2)^{1/5}$ is a so-called chirp mass.

A phenomenological form of the gravitational waveform is given by

$$h(t) = h_0 \cos\phi(t) , \tag{11}$$

where h_0 is called a scaling amplitude and equals to the coefficient of non-zero components of the strain (5)

$$h_0 = \frac{4G\mu}{c^4 d_L} r^2\omega^2 = \frac{4(G\mathcal{M})^{5/3}}{c^4 d_L}\omega^{2/3}. \tag{12}$$

$\phi(t)$ is the gravitational wave phase and it evolves in time as:

$$\phi(t) = 2\omega t + \dot{\omega} t^2 + \phi_0 , \tag{13}$$

where ϕ_0 is the initial phase of binary. Inserting a solution $\omega(t)$ of the differential equation (10) in (12) and (13) one gets a full form of gravitational waveform of merging binary (11).

I built up the code for the software system **Wolfram Mathematica** that does all necessary calculations described above. Carrying out the processes of integration and simplification properly, and solving the differential equation (10), I got the formula for frequency change and for the

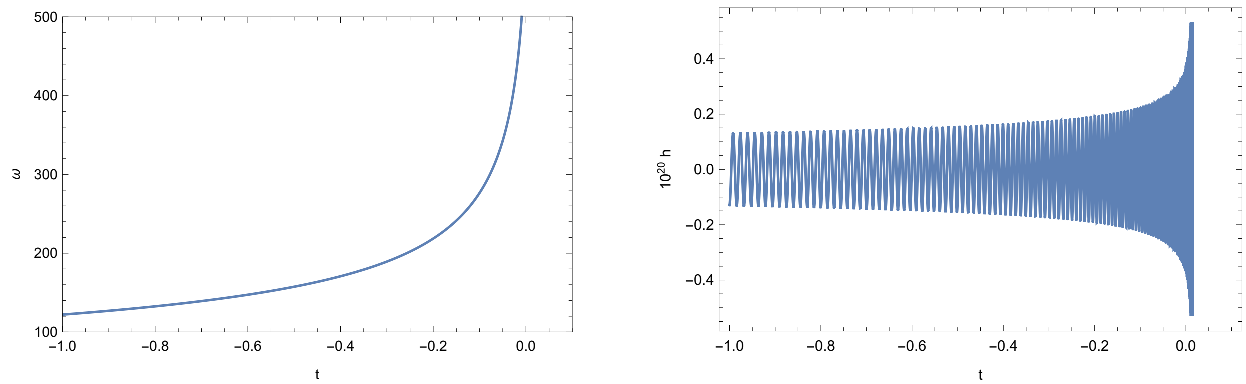

Figure 2: Frequency and gravitational waveform as a function of time

gravitational waveform. The results are presented in the figure (2). The left panel of figure (2) shows the frequency as a function of time and how it increases as time flows and objects get closer to each other. The right panel of figure (2) demonstrates time evolution of gravitational waveform or strain. As stars approach each other frequency and amplitude of the wave increase and at $t = 0$ moment objects merge, ceasing emission of gravitational waves.

The results obtained here are in accordance with the results in the textbooks and papers that discuss the simplified point-mass case. Of course, this is not a full picture. In reality, when objects come close enough with each other, tidal effects are very important and cannot be neglected. Gravitational field gets very strong and one has to add a post-Newtonian expansion terms and calculations get very complicated. In real picture, the time just before the merger looks very different from our simplified discussion. However, this is not important for our purposes; normal star and mirror star does not see each other, so tidal effects won't play an important role. Nevertheless, size of the object would matter as objects will touch each other at different moment. So our goal is to repeat these calculations not for point-mass objects, but for objects that have a finite radius and we want to explore how the evolution of such systems depends on the object's size.

For these purposes, instead of Dirac's delta functions in mass distribution formula (3), we want to use the Heaviside's step function that we build like this:

$$\theta(x, y, z) = \begin{cases} 1, & |r - r_2| < R \\ 0, & |r - r_2| > R \end{cases}, \tag{14}$$

or in more short notation

$$\theta\left(R - \sqrt{(x - x_2)^2 + (y - y_2)^2 + z^2}\right) . \tag{15}$$

This function equals to one, $\theta = 1$, if its argument is > 0, i.e. at distance R from a point $(x_2, y_2, 0)$ and equals to zero in every other place. So it is a ball with radius R centered at some point $(x_2, y_2, 0)$, that for some arbitrary numbers could be drawn using Wolfram Mathematica (figure 3). For simplicity, we consider the occasion when one object has finite radius and second object is still in point-mass approximation. This case could be an example of black hole - neutron star merger, but when one neglects strong field effects, however still accounting for the finite size effects of object. In this set, the mass distribution function (3) will change and take the form:

$$\rho(x, y, z) = m_1\delta(x - x_1)\delta(y - y_1)\delta(z) + m_2 H\left(R - \sqrt{(x - x_2)^2 + (y - y_2)^2 + z^2}\right) . \tag{16}$$

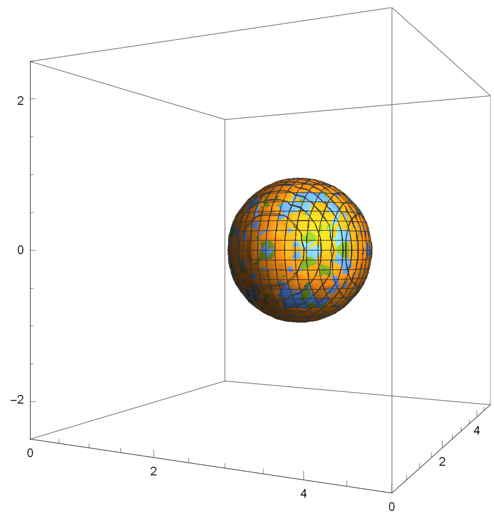

Figure 3: Ilustration of Heaviside's distribution function (15)

But at this point computations do not remain so straightforward. It is not so easy to integrate (16) to get the quadrupole formula. We have checked that, in principle, one can at first differentiate the distribution function with respect to time and do the integration afterwards, without affecting the final results. Such approach looks more accessible in our case. We have made some progress towards this challenging calculations, however some steps still need to be fixed. The work on this task is still ongoing and we hope to have interesting results in the near future.

Once our final formula for frequency change is ready, we will draw the graphs similar to (2), but for 'finite-size case', and compare these two configurations. Knowing the pattern that distinguishes between the finite size case and point-mass approximation, we will be able to search for the possible signs in the waveforms of actual merger events witnessed by LIGO/Virgo detectors. This will enable us to check if mirror world scenario could be plausible for binary compact object mergers, that are seen through gravitational wave detections.

Outcome of the internship: During the internship I started working on a topic that represents one of the sub-themes of my PhD topic. Together with my supervisor Adam Falkowski, we planned to investigate merger of normal star with mirror star and our aim was to search a sign of such mergers in the data of gravitational waves recorded by LIGO/Virgo detectors.

In the first half on the internship, I studied relative literature of gravitational wave physics. After that, I developed the computational code that repeats known calculations for gravitational waveform in point-like approximation. Then we came up with the idea to consider actual size of star, in order to study dependence of merger waveform on the size of object, that is necessary for our case. In mass density distribution formula, instead of Dirac's Delta functions, we inserted Heaviside's step function, that we built up in a proper manner. With this change, computations get quite complicated. We solved many problems in these calculations, however, some steps still need to be fixed before we get our final results.

While dealing with these computations, I managed to advance my skills in Wolfram Mathematica. I mastered myself in dealing with some non-trivial integrals, working with some special functions, solving differential equations and making plots.

Also I attended the international conference "The Paris Saclay AstroParticle Symposium 2021", part of which was devoted to gravitational wave physics. Listening to talks by leading scientists was very helpful for me to get introduced with the latest tools and views in this field.

To summarize, this internship was very productive for me. I developed the ability of working individually; I learned a number of subjects in the field of my research and upgraded my skills in using program Wolfram Mathematica. Overall, the internship was a great experience for me and it will be very useful for completing my PhD thesis.

References

[1] R. Beradze and M. Gogberashvili, "LIGO Signals from the Mirror World," Mon. Not. Roy. Astron. Soc. **487** (2019) no.1, 650-652 doi:10.1093/mnras/stz1295 [arXiv:1902.05425 [gr-qc]].

[2] R. Beradze, M. Gogberashvili and A. S. Sakharov, "Binary Neutron Star Mergers with Missing Electromagnetic Counterparts as Manifestations of Mirror World," Phys. Lett. B **804** (2020), 135402 doi:10.1016/j.physletb.2020.135402 [arXiv:1910.04567 [astro-ph.HE]].

[3] R. Beradze and M. Gogberashvili, "Unexpected LIGO events and the Mirror World," Mon. Not. Roy. Astron. Soc. **503** (2021) no.2, 2882-2886 doi: 10.1093/mnras/stab685 [arXiv:2101.12532 [astro-ph.CO]].

[4] B. P. Abbott *et al.* [LIGO Scientific and Virgo], "The basic physics of the binary black hole merger GW150914," Annalen Phys. **529** (2017) no.1-2, 1600209 doi:10.1002/andp.201600209 [arXiv:1608.01940 [gr-qc]].

[5] Utah State University, Relativistic astrophysics - lecture, Wheeler (2013), http://www.physics.usu.edu/Wheeler/GenRel2013/Notes/GravitationalWaves.pdf

[6] C. Cutler and E. E. Flanagan, "Gravitational waves from merging compact binaries: How accurately can one extract the binary's parameters from the inspiral wave form?," Phys. Rev. D **49** (1994), 2658-2697 doi:10.1103/PhysRevD.49.2658 [arXiv:gr-qc/9402014 [gr-qc]].

[7] A. Guerra Chaves and T. Hinderer, "Probing the equation of state of neutron star matter with gravitational waves from binary inspirals in light of GW170817: a brief review," J. Phys. G **46** (2019) no.12, 123002 doi:10.1088/1361-6471/ab45be [arXiv:1912.01461 [nucl-th]].

[8] B. P. Abbott *et al.* [LIGO Scientific and Virgo], "GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral," Phys. Rev. Lett. **119** (2017) no.16, 161101 doi:10.1103/PhysRevLett.119.161101 [arXiv:1710.05832 [gr-qc]].

[9] D. Radice, S. Bernuzzi and A. Perego, "The Dynamics of Binary Neutron Star Mergers and GW170817," Ann. Rev. Nucl. Part. Sci. **70** (2020), 95-119 doi:10.1146/annurev-nucl-013120-114541 [arXiv:2002.03863 [astro-ph.HE]].

[10] The Paris-Saclay AstroParticle Symposium, 18 October - 26 November 2021, https://indico.ijclab.in2p3.fr/event/7119/